# Attitude Control Final Report

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Section M4A

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Documentation Statement:

# Abstract

To be completed with the project’s summary objectives, methods, and findings.

# Nomenclature

|  |  |  |  |
| --- | --- | --- | --- |
| Symbols | | | |
|  | Amplitude |  |  |
|  | Angular Acceleration |  |  |
|  | Angular Momentum |  |  |
|  | Angular Position |  |  |
|  | Angular Velocity; Crossover Frequency |  |  |
|  | Control Input Matrix |  |  |
|  | Control Signal |  |  |
|  | Coupling Matrix |  |  |
|  | DCM relating CAD body-fixed and Principal Frames |  |  |
|  | Effective Gain |  |  |
|  | Eigen-column |  |  |
|  | Eigenvalue |  |  |
|  | Gain |  |  |
|  | Mass |  |  |
|  | Mean Motion |  |  |
|  | Moment of Inertia |  |  |
|  | Output Matrix |  |  |
|  | Output Vector |  |  |
|  | Position |  |  |
|  | Quaternion |  |  |
|  | Settling Time |  |  |
|  | State Vector |  |  |
|  | System Dynamics Matrix |  |  |
|  | Thruster Firing Delay |  |  |
|  | Time Constant |  |  |
|  | Torque |  |  |
| Subscripts | | | |
|  | Center of Mass |  |  |
|  | Control Input |  |  |
|  | Derivative |  |  |
|  | Matrix index denoting column number. |  |  |
|  | Matrix index denoting row number. |  |  |
|  | Nominal Wheel Speed |  |  |
|  | Payload |  |  |
|  | Proportional |  |  |
|  | Reaction Wheel |  |  |
|  | Relating to the CAD Body-Fixed Frame |  |  |
|  | Relating to the Orbital Frame |  |  |
|  | Relating to the Principal Frame |  |  |
|  | Spacecraft |  |  |

# Introduction

The objective of this project is to design and analyze an attitude control system for FalconSAT-9, a next-generation small satellite funded by the Air Force Research Laboratory (AFRL). The spacecraft will conduct advanced propulsion and space maneuvering experiments using an experimental Hall-effect thruster. FalconSAT-9 will be deployed into a 500 km altitude polar orbit as a secondary payload on a Falcon 9 launch vehicle. Upon deployment, the primary mission phases include detumbling the satellite and achieving precise orbit-fixed orientation during propulsion experiments.

The attitude determination and control system (ADCS) must counteract initial angular rates, environmental disturbances, and constant torques induced by propulsion misalignment. The system integrates multiple control modes to meet the spacecraft's mission objectives: Control Mode 0 focuses on detumbling, while Control Mode 1 ensures precise attitude during propulsion experiments. This report outlines the theoretical analysis, control design, and experimental validation necessary to meet these requirements.

# Theory

### Assumptions

The analysis assumes the satellite operates under idealized conditions, including:

1. A rigid-body spacecraft with mass properties derived from CAD body-fixed and principal frames.
2. The Earth is modeled as a perfect sphere with uniform density.
3. External disturbance torques include gravity gradient, atmospheric drag, and magnetic torques; solar pressure is negligible.

### Mathematical Techniques

First, the necessary calculations are made to obtain the spacecraft’s mass properties. We start by assuming the spacecraft consists of two parts: the body and the payload. The given physical parameters used for the following calculations are included in Appendix ##. The total center of mass (COM) of the spacecraft is derived by treating the components as point masses located at their respective COM positions in the CAD frame. We then multiply each component's mass by its position vector to compute its contribution to the total moment about the origin, sum these moments for all components, and divide by the total mass to determine the spacecraft’s overall COM.

|  |  |
| --- | --- |
|  | (1) |

Using the spacecraft’s COM, adjusting the offset position vectors by the COM vector. Using the given inertia matrices for the body and the payload, we apply the linearized parallel axis theorem,

|  |  |
| --- | --- |
|  | (2) |

The resulting moment of inertia matrices are then added together for the total moment of inertia of the satellite in the CAD frame.

|  |  |
| --- | --- |
|  | (3) |

Now we need a way to relate the CAD frame to the principal frame. To find the directional cosine matrix that relates the spacecraft's body frame to a principal frame, solve for the eigenvalues and eigenvectors of the inertia matrix in the body frame.

|  |  |
| --- | --- |
|  | (4) |

Arranging the eigenvectors as the columns of a 3x3 matrix such that the resulting matrix is close to identity gives the DCM .

|  |  |
| --- | --- |
|  | (5) |

Taking the transpose of gives . Because any scalar multiple of an eigenvector is also an eigenvector corresponding to the same eigenvalue, multiply by -1 as necessary to get close to the identity matrix. Then, pre and post multiply by the DCM to get the moment of inertia matrix in the principal frame.

|  |  |
| --- | --- |
|  | (6) |

### Control Mode 1

To develop and apply Linear-Quadratic control techniques, a linearized model of the nonlinear equations of motion must be derived. The equations of motion (EOM) for the spacecraft are derived in the orbital local vertical/local horizontal (LVLH) reference frame, accounting for gravity gradient torque and reaction wheel control, with the LVLH frame origin at the spacecraft's center of mass. The nonlinear equations modeling spacecraft dynamics with gravity gradient torques, reaction wheels, using quaternions are:

|  |  |
| --- | --- |
|  | (7) |

The state vector is defined as,

|  |  |
| --- | --- |
|  | (8) |

The linearized state-space form is defined as:

|  |  |
| --- | --- |
|  | (7) |

To linearize the EOMs, we take the Jacobian with respect to the state vector and control input for the A and B matrices respectively, where the control input is the control torque vector.

|  |  |
| --- | --- |
|  | (8) |

The C matrix is a 9 by 9 identity matrix, and the coupling matrix D is zero. Note that there are 10 distinct states: 3 relating to angular velocity between the frames, 4 relating to the quaternion describing the relationship between the two frames, and 3 relating to the angular momentum of the spacecraft. Because the quaternion is defined to be unit in length and the satellite is pointing downward (along one of the three axes in the LVLH frame), only three components of the quaternion are required to fully describe the relationship between the body frame and the orbital frame. Specifically,  is no longer required, and therefore the number of states decreases by one.

Taking the partial derivatives of each of the equations with respect to each of the states yields a 10 by 9 matrix containing the partial derivatives of each equation with respect to each state. Each row represents an equation, and each column represents a different variable with respect to which the equation was derived.

Following the determination of the A and B matrices, linearizing about the equilibrium point where the satellite is pointing down in the LVLH frame yields,

|  |  |
| --- | --- |
|  | (8) |

Included in Appendix ## is the fully evaluated, linearized state-space form of the A, B, C, and D matrices.

### Control Mode 0

The spacecraft is initially in a tumble and therefore and control mode is required to detumble it. Micro-thrusters are modeled with on-off (bang-bang) dynamics to detumble the spacecraft.

The detumbling process in Control Mode 0 is achieved by leveraging nonlinear control techniques applied to the spacecraft's pitch dynamics. The thrusters are modeled as ideal on-off actuators, providing constant positive or negative torque depending on the control signal.

|  |  |
| --- | --- |
|  | (8) |

Here, is the thruster torque, and is the control signal. This introduces relay-like behavior, which necessitates nonlinear control analysis. The system dynamics are simplified by isolating pitch motion while assuming no coupling with roll and yaw dynamics. The equations of motion are derived using rigid-body dynamics, establishing a second-order differential equation that relates the pitch angle, angular velocity, and thruster torque.

|  |  |
| --- | --- |
|  | (8) |

is the spacecraft’s moment of inertia about the pitch axis, is the angular acceleration, and is the applied torque. Phase plane analysis is employed to understand the spacecraft's behavior. The angular velocity and pitch angle are linked to the system's total energy. The relationship is generally as,

|  |  |
| --- | --- |
|  | (8) |

The derived energy function is used to plot trajectories in the phase plane, where the horizontal axis represents the pitch angle, and the vertical axis represents angular velocity. These trajectories are categorized based on thruster torque: no torque, positive torque, and negative torque. The critical energy boundary separating oscillatory (libration) and tumbling motion is identified through this analysis.

The control strategy is implemented using a proportional control law, where the control signal is proportional to the pitch angle, which follows as,

|  |  |
| --- | --- |
|  | (8) |

This creates a switching condition in the phase plane, enabling the system to toggle between positive and negative torques. Because the objective is to drive the spacecraft orientation so that the principal axis body frame is aligned with the orbital frame, a proportional-derivative feedback law is implemented, adding a term proportional to angular velocity.

|  |  |
| --- | --- |
|  | (8) |

This adjustment defines a switching line in the phase plane with a slope determined by the ratio of proportional to derivative gains, guiding the pitch angle and angular velocity to zero. Along the switching line, the system exhibits first-order behavior,

|  |  |
| --- | --- |
|  | (8) |

Which has the solution,

|  |  |
| --- | --- |
|  | (8) |

From here we can pull out the time constant and resulting settling time approximation as a result of first-order behavior,

|  |  |
| --- | --- |
|  | (8) |

The nonlinear nature of the system introduces control chattering, resulting in a limit cycle during rapid thruster switching. To mitigate the limit cycle, a deadband is introduced around the switching line, reducing excessive thruster toggling.

|  |  |
| --- | --- |
|  | (8) |

Where is the width of the deadband. The deadband is designed to lower the effective gain below the gain margin, ensuring stability while maintaining control accuracy.

Describing function analysis is used to approximate this nonlinearity as a frequency-dependent gain, enabling stability analysis in the frequency domain. This technique is crucial for assessing the system's behavior, particularly to predict and mitigate limit cycles, which are persistent oscillations caused by interactions between the relay and the system dynamics. The relay generates a nonlinear relationship between the input control signal and the output torque. To approximate this behavior, it is assumed that the system input is a sinusoidal signal of amplitude and frequency . The relay output is then represented as a square wave with fixed amplitude and the same frequency. The relay is thus modeled as an equivalent gain:

|  |  |
| --- | --- |
|  | (8) |

where is the maximum torque, and is the amplitude of the sinusoidal input. This describing function is inversely proportional to the input amplitude, meaning the gain decreases as the input amplitude increases. The describing function is then incorporated into the system's open-loop transfer function, which represents the linear dynamics of the system. The closed-loop stability condition is expressed as:

|  |  |
| --- | --- |
|  | (8) |

where is the magnitude of the transfer function. This condition identifies the amplitude and frequency at which the system becomes marginally stable, indicating the onset of a limit cycle. To further understand these oscillations, the critical amplitude and frequency are derived. The amplitude of the limit cycle is calculated using:

|  |  |
| --- | --- |
|  | (8) |

Which relates the oscillation amplitude to the describing function gain. The frequency at which these oscillations occur, known as the crossover frequency, is determined by the phase lag introduced by actuator time delays:

|  |  |
| --- | --- |
|  | (8) |

where is the delay time of the thruster actuation. The phase lag grows with increasing frequency, and when it reaches the system exhibits a limit cycle. These analyses are essential for understanding the stability of the nonlinear control system. By tuning the deadband or modifying controller gains, the system's stability and performance are ensured, allowing for effective detumbling of the spacecraft.

# Theoretical Predictions

This section will be completed with the outcomes of the theoretical development once the calculations are finalized.

# Experimental Results

This section will present experimental or computational findings. To be completed post-data collection.

# Discussion

This section will analyze the results, compare experimental data with theoretical predictions, and discuss discrepancies or sources of error.

# Conclusions and Recommendations

To be completed with a concise summary of findings and recommendations for future work.

# Appendices

Include supplementary materials, calculations, and MATLAB Simulink block diagrams here.