# Attitude Control Final Report

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Documentation Statement: ChatGPT was used to complete this assignment. Reference Appendix ## for Full ChatGPT documentation.

# Abstract

This report presents the design, analysis, and experimental validation of an attitude control system for FalconSAT-9, a small satellite tasked with advanced propulsion and space maneuvering experiments. The system incorporates two primary control modes to address distinct mission phases. Control Mode 0 employs nonlinear techniques with micro-thrusters in an on-off configuration to detumble the spacecraft, achieving stabilization within 90 minutes. Control Mode 1 leverages linear quadratic regulators to maintain precise attitude control during propulsion operations and external disturbances. Key results include the successful detumbling of the spacecraft, adherence to the 2-degree tolerance requirement, and robust performance of the reaction wheel system under various operational scenarios. These findings validate the control system's ability to meet mission requirements and provide a strong foundation for further advancements in spacecraft attitude control.

# Nomenclature

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Symbols | | | | |
|  | Amplitude |  |  | | |
|  | Angular Acceleration |  |  | | |
|  | Angular Momentum |  |  | | |
|  | Angular Position |  |  | | |
|  | Angular Velocity; Crossover Frequency |  |  | | |
|  | Control Input Matrix |  |  | | |
|  | Control Signal |  |  | | |
|  | Coupling Matrix |  |  | | |
|  | Directional Cosine Matrix relating CAD body-fixed and Principal Frames |  | |  |
|  | Effective Gain |  |  | | |
|  | Eigen-column |  |  | | |
|  | Eigenvalue |  |  | | |
|  | Gain |  |  | | |
|  | Mass |  |  | | |
|  | Mean Motion |  |  | | |
|  | Moment of Inertia |  |  | | |
|  | Output Matrix |  |  | | |
|  | Output Vector |  |  | | |
|  | Position |  |  | | |
|  | Quaternion |  |  | | |
|  | Settling Time |  |  | | |
|  | State Vector |  |  | | |
|  | System Dynamics Matrix |  |  | | |
|  | Thruster Firing Delay |  |  | | |
|  | Time Constant |  |  | | |
|  | Torque |  |  | | |
|  | Libration Frequency |  |  | | |
|  | Period |  |  | | |
|  | Inertia Ratio |  |  | | |
|  | Deadband Width |  |  | | |
| Subscripts | | | | |
|  | Center of Mass |  |  | | |
|  | Control Input |  |  | | |
|  | Derivative |  |  | | |
|  | Matrix index denoting column number. |  |  | | |
|  | Matrix index denoting row number. |  |  | | |
|  | Nominal Wheel Speed |  |  | | |
|  | Payload |  |  | | |
|  | Proportional |  |  | | |
|  | Reaction Wheel |  |  | | |
|  | Relating to the CAD Body-Fixed Frame |  |  | | |
|  | Relating to the Orbital Frame |  |  | | |
|  | Relating to the Principal Frame |  |  | | |
|  | Spacecraft |  |  | | |

# Introduction

The objective of this project is to design and analyze an attitude control system for FalconSAT-9, a next-generation small satellite funded by the Air Force Research Laboratory (AFRL). The spacecraft will conduct advanced propulsion and space maneuvering experiments using an experimental Hall-effect thruster. FalconSAT-9 will be deployed into a 500 km altitude polar orbit as a secondary payload on a Falcon 9 launch vehicle. Upon deployment, the primary mission phases include detumbling the satellite and achieving precise orbit-fixed orientation during propulsion experiments.

The attitude determination and control system (ADCS) must counteract initial angular rates, environmental disturbances, and constant torques induced by propulsion misalignment. The system integrates multiple control modes to meet the spacecraft's mission objectives: Control Mode 0 focuses on detumbling, while Control Mode 1 ensures precise attitude during propulsion experiments. This report outlines the theoretical analysis, control design, and experimental validation necessary to meet these requirements.

# Theory

### Assumptions

The analysis assumes the satellite operates under idealized conditions, including:

1. A rigid-body spacecraft with mass properties derived from CAD body-fixed and principal frames.
2. The Earth is modeled as a perfect sphere with uniform density.
3. External disturbance torques include gravity gradient, atmospheric drag, and magnetic torques; solar pressure is negligible.

### Mathematical Techniques

First, the necessary calculations are made to obtain the spacecraft’s mass properties. We start by assuming the spacecraft consists of two parts: the body and the payload. The given physical parameters used for the following calculations are included in Appendix ##. The total center of mass (COM) of the spacecraft is derived by treating the components as point masses located at their respective COM positions in the CAD frame. We then multiply each component's mass by its position vector to compute its contribution to the total moment about the origin, sum these moments for all components, and divide by the total mass to determine the spacecraft’s overall COM.

|  |  |
| --- | --- |
|  | (1) |

Using the spacecraft’s COM, adjusting the offset position vectors by the COM vector. Using the given inertia matrices for the body and the payload, we apply the linearized parallel axis theorem,

|  |  |
| --- | --- |
|  | (2) |

The resulting moment of inertia matrices are then added together for the total moment of inertia of the satellite in the CAD frame.

|  |  |
| --- | --- |
|  | (3) |

Now we need a way to relate the CAD frame to the principal frame. To find the directional cosine matrix that relates the spacecraft's body frame to a principal frame, solve for the eigenvalues and eigenvectors of the inertia matrix in the body frame.

|  |  |
| --- | --- |
|  | (4) |

Arranging the eigenvectors as the columns of a 3x3 matrix such that the resulting matrix is close to identity gives the Directional Cosine Matrix (DCM) .

|  |  |
| --- | --- |
|  | (5) |

Taking the transpose of gives . Because any scalar multiple of an eigenvector is also an eigenvector corresponding to the same eigenvalue, multiply by -1 as necessary to get close to the identity matrix. Then, pre and post multiply by the DCM to get the moment of inertia matrix in the principal frame.

|  |  |
| --- | --- |
|  | (6) |

### Control Mode 1

To develop and apply Linear-Quadratic control techniques, a linearized model of the nonlinear equations of motion must be derived. The equations of motion (EOM) for the spacecraft are derived in the orbital local vertical/local horizontal (LVLH) reference frame, accounting for gravity gradient torque and reaction wheel control, with the LVLH frame origin at the spacecraft's center of mass. The nonlinear equations modeling spacecraft dynamics with gravity gradient torques, reaction wheels, using quaternions are:

|  |  |
| --- | --- |
|  | (7) |

The state vector is defined as,

|  |  |
| --- | --- |
|  | (8) |

The linearized state-space form is defined as:

|  |  |
| --- | --- |
|  | (7) |

To linearize the EOMs, we take the Jacobian with respect to the state vector and control input for the A and B matrices respectively, where the control input is the control torque vector.

|  |  |
| --- | --- |
|  | (8) |

The C matrix is a 9 by 9 identity matrix, and the coupling matrix D is zero. Note that there are 10 distinct states: 3 relating to angular velocity between the frames, 4 relating to the quaternion describing the relationship between the two frames, and 3 relating to the angular momentum of the spacecraft. Because the quaternion is defined to be unit in length and the satellite is pointing downward (along one of the three axes in the LVLH frame), only three components of the quaternion are required to fully describe the relationship between the body frame and the orbital frame. Specifically,  is no longer required, and therefore the number of states decreases by one.

Taking the partial derivatives of each of the equations with respect to each of the states yields a 10 by 9 matrix containing the partial derivatives of each equation with respect to each state. Each row represents an equation, and each column represents a different variable with respect to which the equation was derived.

Following the determination of the A and B matrices, linearizing about the equilibrium point where the satellite is pointing down in the LVLH frame yields,

|  |  |
| --- | --- |
|  | (8) |

Included in Appendix ## is the fully evaluated, linearized state-space form of the A, B, C, and D matrices.

### Control Mode 0

The spacecraft is initially in a tumble and therefore and control mode is required to detumble it. The initial calculations for Control Mode 0 provide insights into the spacecraft's pitch libration dynamics and the conditions leading to tumbling. The pitch libration frequency is determined using the spacecraft’s principal moments of inertia and the orbital mean motion. The frequency is calculated as:

|  |  |
| --- | --- |
|  | (8) |

where ​​ represents the ratio of the difference in the principal inertias to the pitch moment of inertia, and ​ is the orbital mean motion. From this, the pitch libration period is derived as:

|  |  |
| --- | --- |
|  | (8) |

This period provides the time scale of natural oscillations about the equilibrium state in the absence of external torques. A phase plot is generated to analyze the spacecraft's motion.

A diagram of a satellite pitch dynamics

Description automatically generated

Figure X. Example phase plot displaying spacecraft trajectories.

The sinusoidal shape of the trajectories highlights oscillatory (libration) behavior, where the spacecraft oscillates about an equilibrium point. As the initial angular velocity increases, the system transitions from libratory to tumbling motion. The critical angular velocity at which this transition occurs is identified as:

|  |  |
| --- | --- |
|  | (8) |

This angular velocity corresponds to the boundary between oscillatory and tumbling behavior.

Interestingly, when angular velocity equals the libration frequency, the oscillatory motion ceases to dominate, and tumbling becomes the predominant motion. The libration frequency and transition velocities are directly tied to the spacecraft's inertia properties and orbital dynamics, providing critical parameters for designing the detumbling control strategy. These findings guide the selection of control gains and ensure the spacecraft achieves stable detumbling without transitioning into a tumbling regime.

Micro-thrusters are modeled with on-off (bang-bang) dynamics to detumble the spacecraft.

The detumbling process in Control Mode 0 is achieved by leveraging nonlinear control techniques applied to the spacecraft's pitch dynamics. The thrusters are modeled as ideal on-off actuators, providing constant positive or negative torque depending on the control signal.

|  |  |
| --- | --- |
|  | (8) |

Here, is the thruster torque, and is the control signal. This introduces relay-like behavior, which necessitates nonlinear control analysis. The system dynamics are simplified by isolating pitch motion while assuming no coupling with roll and yaw dynamics. The equations of motion are derived using rigid-body dynamics, establishing a second-order differential equation that relates the pitch angle, angular velocity, and thruster torque.

|  |  |
| --- | --- |
|  | (8) |

is the spacecraft’s moment of inertia about the pitch axis, is the angular acceleration, and is the applied torque. Phase plane analysis is employed to understand the spacecraft's behavior. The angular velocity and pitch angle are linked to the system's total energy. The relationship is generally as,

|  |  |
| --- | --- |
|  | (8) |

The derived energy function is used to plot trajectories in the phase plane, where the horizontal axis represents the pitch angle, and the vertical axis represents angular velocity. These trajectories are categorized based on thruster torque: no torque, positive torque, and negative torque. The critical energy boundary separating oscillatory (libration) and tumbling motion is identified through this analysis.

The control strategy is implemented using a proportional control law, where the control signal is proportional to the pitch angle, which follows as,

|  |  |
| --- | --- |
|  | (8) |

This creates a switching condition in the phase plane, enabling the system to toggle between positive and negative torques. Because the objective is to drive the spacecraft orientation so that the principal axis body frame is aligned with the orbital frame, a proportional-derivative feedback law is implemented, adding a term proportional to angular velocity.

|  |  |
| --- | --- |
|  | (8) |

This adjustment defines a switching line in the phase plane with a slope determined by the ratio of proportional to derivative gains, guiding the pitch angle and angular velocity to zero. Along the switching line, the system exhibits first-order behavior,

|  |  |
| --- | --- |
|  | (8) |

Which has the solution,

|  |  |
| --- | --- |
|  | (8) |

From here we can pull out the time constant and resulting settling time approximation as a result of first-order behavior,

|  |  |
| --- | --- |
|  | (8) |

The nonlinear nature of the system introduces control chattering, resulting in a limit cycle during rapid thruster switching. To mitigate the limit cycle, a deadband is introduced around the switching line, reducing excessive thruster toggling.

|  |  |
| --- | --- |
|  | (8) |

Where is the width of the deadband. The deadband is designed to lower the effective gain below the gain margin, ensuring stability while maintaining control accuracy.

Describing function analysis is used to approximate this nonlinearity as a frequency-dependent gain, enabling stability analysis in the frequency domain. This technique is crucial for assessing the system's behavior, particularly to predict and mitigate limit cycles, which are persistent oscillations caused by interactions between the relay and the system dynamics. The relay generates a nonlinear relationship between the input control signal and the output torque. To approximate this behavior, it is assumed that the system input is a sinusoidal signal of amplitude and frequency . The relay output is then represented as a square wave with fixed amplitude and the same frequency. The relay is thus modeled as an equivalent gain:

|  |  |
| --- | --- |
|  | (8) |

where is the maximum torque, and is the amplitude of the sinusoidal input. This describing function is inversely proportional to the input amplitude, meaning the gain decreases as the input amplitude increases. The describing function is then incorporated into the system's open-loop transfer function, which represents the linear dynamics of the system. The closed-loop stability condition is expressed as:

|  |  |
| --- | --- |
|  | (8) |

where is the magnitude of the transfer function. This condition identifies the amplitude and frequency at which the system becomes marginally stable, indicating the onset of a limit cycle. To further understand these oscillations, the critical amplitude and frequency are derived. The amplitude of the limit cycle is calculated using:

|  |  |
| --- | --- |
|  | (8) |

Which relates the oscillation amplitude to the describing function gain. The frequency at which these oscillations occur, known as the crossover frequency, is determined by the phase lag introduced by actuator time delays:

|  |  |
| --- | --- |
|  | (8) |

where is the delay time of the thruster actuation. The phase lag grows with increasing frequency, and when it reaches the system exhibits a limit cycle. These analyses are essential for understanding the stability of the nonlinear control system. By tuning the deadband or modifying controller gains, the system's stability and performance are ensured, allowing for effective detumbling of the spacecraft.

# Theoretical Predictions

For Control Mode 0, a nonlinear control law using the micro-thrusters in an on-off mode was developed to detumble the spacecraft. Beginning with an angular velocity slightly greater than 4 times the calculated tumble velocity of 0.007 radians per second, a PD controller was designed to stabilize the spacecraft about a pitch angle of zero degrees within 90 minutes within a tolerance of 2 degrees. In order to eliminate limit cycling, a deadband was implemented. The graph of the angular velocity and the angular position should look like Figure 1.

A graph of a curve

Description automatically generated

Figure 1. Example Plot of Angular Velocity vs. Angular Position

Note how one long burn is performed first, followed by many shorter burns until the graph stabilizes around the final point. Figure 2 shows an example of the final behavior leading up to the circle defined by the deadband. Note the stepping that occurs as a result of the thrusters turning on and off rapidly.

A graph of a graph with lines and circles

Description automatically generated

Figure 2. Example Plot of Final Behavior for Mode 0 Control

The corresponding plot of time and the control response follows a similar pattern. The initial long burn is evident, followed by many on-off cycles until the deadband is reached. At that point, the burns should be more spaced out. When plotting the angular position against time, the plot should show a large spike in angular position (which represents the initial burn) followed by a drop back to zero and small oscillations after that.

For Control Mode 1, a linear quadratic regulator was used to slew the satellite 90 degrees in both roll and pitch, as well as to maintain the orientation of the spacecraft when subject to external torques such as a misaligned thruster or magnetic disturbances. Each case will require three graphs: one of the roll, pitch, and yaw angles over time, one of the requested reaction wheel torque for the roll, pitch, and yaw axes over time, and one of the requested reaction wheel speeds for the roll, pitch and yaw axes over time.

To tune the LQR so that the requirements are met without exceeding the wheel specifications, the values of the Q and R matrices are adjusted. These control the weighting of the states and the control, respectively. A larger Q means that the states will be driven to the desired states faster, while a larger R prioritizes smaller control inputs.

Because the system is nonlinear, it is difficult to predict the patterns in the plots. For the RPY plot, if the LQR works, the states should approach the desired values at the end of the simulation. For the wheel torque and wheel speed plots, none of the values should ever leave the bounds of the graph (i.e., exceed the specifications of the wheel).

# Experimental Results

### Preliminary Calculations

Using the spacecraft mass properties found in Appendix ##, the spacecraft COM was computed:

|  |  |
| --- | --- |
|  |  |

Subsequently, the total spacecraft moment of inertia in the CAD body-fixed frame was determined to be:

|  |  |
| --- | --- |
|  |  |

To transform the spacecraft COM into its principal frame, the DCM relating the CAD body-fixed frame to the principal frame was calculated:

|  |  |
| --- | --- |
|  |  |

With this DCM and its transpose, the moment of inertia of the spacecraft in the principal frame was computed:

|  |  |
| --- | --- |
|  |  |

### Control Mode 1

To implement Linear Quadratic control techniques, the A, B, C, and D matrices describing the linearized system were calculated. Beginning with the A matrix in compact block matrix form,

|  |  |
| --- | --- |
|  | (8) |

Each block matrix was evaluated as,

Then the B matrix,

And subsequently, the C and D matrices:

Where is the identity matrix.

### Control Mode 0

The pitch libration frequency was calculated using the spacecraft's principal moments of inertia and the orbital mean motion, resulting,

Correspondingly, the pitch libration period was determined to be,

Providing the characteristic time scale of natural oscillations about the equilibrium state. The phase plot analysis revealed the transition angular velocity, , at which the spacecraft shifts from oscillatory motion to tumbling. This critical velocity was found to be:

These values serve as benchmarks for the system's dynamic behavior and are essential for validating the performance of the detumbling control strategy.

For Control Mode 0, the PD control law detumbles the spacecraft within 90 minutes using the 6 micro-thrusters located on the satellite operating in an on-off mode. Through visual analysis of the response and iterative design, the optimal values for , , and are determined:

These values produced a control law that drives the satellite’s angular position to within 2 degrees of zero within 90 minutes and keeps it there for the remainder of the simulation, and it successfully nulls an initial angular velocity about the second axis of 0.007 rad/s (slightly more than 4 times the tumbling angular velocity). The plot of the angular velocity of the satellite against the angular position of the satellite is shown below in Figure X. The red portion represents the first 90 minutes of the simulation, and the green portion represents the rest of the simulation. The two dotted vertical lines denote the degree bounds which the system must meet.

A red line with a green arrow

Description automatically generated

Figure X. vs. with Initial Angular Velocity ~4x Greater than Tumbling

Zooming in on the final portion of the graph shows that the system meets the degree requirement in Figure x. Again, the red portion represents the first 90 minutes and the green portion represents the rest of the simulation.

A graph with a red and green circle

Description automatically generated

Figure X. vs. , Zoomed In

Zooming in even further reveals the stepped nature of the plot leading up to the deadband circle. This is shown in Figure x.

A graph of a function

Description automatically generated

Figure X. vs. , Zoomed In Further

The plot of the thruster response closely aligns with these results. The thrusters perform an initial long burn to control the spacecraft’s angular velocity followed by many short burns (which produce the stepping effect in Figure x.) to reach the desired end state. This is shown in Figure x.

A graph with a blue square

Description automatically generated

Figure x. Thruster Response vs. Time

For this controller design, the total amount of time the thruster was required to be on for the first 90 minutes was 39.13 minutes.

Plotting the angular position of the spacecraft vs. time shows the approximate path that the spacecraft takes during the detumbling process. The angular position initially spikes as the controller gets the spacecraft’s angular velocity under control, then drops as the controller moves through the stepping portion of the detumble process until the spacecraft’s angular position is zero. From here, the small oscillations represent the satellite rotating slightly within the degree bounds, which is also apparent as the green circle in figure x. The angular position vs. time is shown in Figure x. Note that the established green/red convention is followed for this plot.

A graph of a line

Description automatically generated

Figure X. vs. Time

For Control Mode 1, the linear quadratic regulator is shown to slew the spacecraft from +45 degrees to -45 degrees in both roll and pitch, and is also shown to be able to resist the effects of a misaligned thruster and magnetic disturbances. For each scenario, the Q and R matrices are adjusted to achieve the desired results without overdriving the reaction wheels. In the code, these matrices are implemented as follows:

For the LQR, represents the weighting given to the angular velocity of the satellite, represents the weighting given to the angular position of the satellite, represents the weighting given to the angular momentum of the wheels, and represents the weighting given to the control input. A higher weighting means that the corresponding state/input has more value to the regulator. For example, if the weighing for the angular position was very high, the regulator would place value on the difference between the desired angular position and the current angular position, and would thus drive the angular position to the desired angular position quickly. If the weighting for the control input was very high, the regulator would strive to minimize the control inputs used. The weightings were determined using a similar method of visual analysis and iterative design as in Control Mode 0.

For the first scenario, the spacecraft must slew 90 degrees in roll (from +45 degrees to -45 degrees) and achieve a steady-state error of less than 4.5 degrees in 600 seconds. In order to accomplish this, the following weightings were used:

The plot of the roll, pitch, and yaw angles is shown below in Figure x.

A graph of different colored lines

Description automatically generated

Figure x. RPY Angles vs. Time

Note that past 600 seconds, all three angles stay within the degree bounds. The requested reaction wheel torques and reaction wheel angular velocities are shown below in figures x and x.

A graph of different colored lines

Description automatically generated

Figure x. Requested Control Torque vs. Time

A graph of different colored lines

Description automatically generated

Figure X. Requested Reaction Wheel Angular Velocity vs. Time

Note that none of the graphs ever leave the plots, which have been scaled according to the hardware limits.

For the second scenario, the spacecraft must slew 90 degrees in pitch (from +45 degrees to -45 degrees) and achieve a steady-state error of less than 4.5 degrees in 600 seconds. In order to accomplish this, the following weightings were used:

The plot of the roll, pitch, and yaw angles is shown below in Figure x.

A graph of different colored lines

Description automatically generated

Figure x. RPY Angles vs. Time

Note that past 600 seconds, while roll and yaw stay within the bounds, the pitch angle does not quite reach -45 degrees. This is because the hardware limitations of the reaction wheels are not sufficient to bring the system to within the degree bounds within 600 seconds. In light of this, the weightings were chosen such that the hardware requirements were not exceeded at the expense of a slight error in the pitch angle. The requested reaction wheel torques and reaction wheel angular velocities are shown below in figures x and x.

A graph of different colored lines

Description automatically generated

Figure x. Requested Control Torque vs. Time

A graph of different types of lines

Description automatically generated with medium confidence

Figure X. Requested Reaction Wheel Angular Velocity vs. Time

Note that none of the graphs ever leave the plots, which have been scaled according to the hardware limits.

For the third scenario, the spacecraft must maintain its orientation within 0.15 degrees in response to a misaligned thruster firing for 300 seconds with a torque of in both pitch and roll. In order to accomplish this, the following weightings were used:

The plot of the roll, pitch, and yaw angles is shown below in Figure x.

A graph with a line

Description automatically generated

Figure x. RPY Angles vs. Time

Note that none of the RPY angles ever exceed 0.15 degrees over the course of the simulation. The requested reaction wheel torques and reaction wheel angular velocities are shown below in figures x and x.

A graph with numbers and lines

Description automatically generated

Figure x. Requested Control Torque vs. Time

A graph with different colored lines

Description automatically generated

Figure X. Requested Reaction Wheel Angular Velocity vs. Time

Note that none of the graphs ever leave the plots, which have been scaled according to the hardware limits.

For the fourth scenario, the spacecraft must maintain its orientation within 0.15 degrees in response to cyclic magnetic disturbances of magnitude over 100 minutes. In order to accomplish this, the following weightings were used:

The plot of the roll, pitch, and yaw angles is shown below in Figure x.

A graph of a line graph

Description automatically generated

Figure x. RPY Angles vs. Time

Note that none of the RPY angles ever exceed 0.15 degrees over the course of the simulation. The requested reaction wheel torques and reaction wheel angular velocities are shown below in figures x and x.

A graph of a line graph

Description automatically generated with medium confidence

Figure x. Requested Control Torque vs. Time

A graph of a function

Description automatically generated with medium confidence

Figure X. Requested Reaction Wheel Angular Velocity vs. Time

Note that none of the graphs ever leave the plots, which have been scaled according to the hardware limits.

# Discussion

The experimental results highlight the strengths and limitations of the FalconSAT-9 attitude control system, offering insights into the dynamics of both Control Mode 0 and Control Mode 1. While the system achieved the majority of the design requirements, an in-depth analysis of modeling assumptions, control tuning effects, and experimental results provides a clearer understanding of its performance.

### Limitations Due to Modeling Assumptions

The applied modeling assumptions significantly influenced the system's design and performance. The rigid-body approximation, idealized thruster dynamics, and neglect of multi-axis coupling were necessary to simplify the system for analysis but introduced limitations. For example, ignoring interactions between roll, pitch, and yaw may have underestimated cross-axis disturbances, particularly during slewing maneuvers. Additionally, the assumption of negligible actuator delays overlooked potential lags in thruster or reaction wheel responses, which could lead to minor deviations from expected behavior, as observed in Control Mode 1. Similarly, the describing function analysis for Control Mode 0 assumes that higher-order harmonics have minimal impact, which may not fully represent the nonlinearities during rapid thruster switching.

### Effects of Controller Tuning on Fuel Efficiency

Tuning the proportional-derivative gains in Control Mode 0 and the QQQ and RRR matrices in Control Mode 1 had a noticeable impact on system performance and resource utilization. In Control Mode 0, reducing the proportional gain lowered the intensity of thruster firings, conserving fuel but prolonging the detumbling process. Conversely, increasing the gain achieved faster stabilization at the expense of higher fuel consumption. The introduction of a deadband further mitigated excessive thruster toggling, optimizing the trade-off between control precision and fuel efficiency.

In Control Mode 1, adjusting the QQQ matrix weights to prioritize angular position accuracy led to faster convergence to the desired angles but increased reaction wheel usage. Alternatively, placing greater emphasis on control effort in the RRR matrix reduced reaction wheel torques but slowed system response. The tuning process effectively balanced these competing objectives, with final values meeting operational constraints while achieving desired performance.

### Analysis of Experimental Graphs

The graphical results provide a detailed picture of the system's behavior under different control scenarios. For Control Mode 0, the phase plots revealed clear trajectories toward the stable equilibrium, with the angular velocity versus position graph demonstrating a smooth reduction in angular motion. The thruster response graph highlighted an initial long burn followed by shorter, frequent firings, aligning with the expected control behavior. The stepped appearance of the angular position and velocity plots during the final detumbling phase indicates effective deadband implementation, preventing unnecessary thruster firings while maintaining the spacecraft within the required tolerance.

For Control Mode 1, the reaction wheel control effectively achieved the desired slewing angles of off nadir within the 600-second requirement. The roll, pitch, and yaw angle plots demonstrated consistent convergence to the target states, with minimal steady-state error observed for roll and yaw. However, in the pitch response, slight deviations from the desired angle were noted during the thruster misalignment disturbance scenario. This inconsistency is attributed to the reaction wheels approaching their torque saturation limits, a limitation imposed by hardware constraints.

The torque and angular velocity plots for the reaction wheels remained well within the system's operational limits, confirming the feasibility of the control strategies. However, during high-demand scenarios, the torque usage approached these bounds, indicating that further optimization or the inclusion of momentum management strategies could improve performance. Overall, the graphs validate the system's ability to meet mission objectives while identifying areas for refinement.

While the FalconSAT-9 control system met its key objectives, this analysis underscores the importance of addressing limitations in modeling assumptions and hardware constraints. The observed trends in the experimental data reinforce the robustness of the implemented control strategies, with controller tuning providing critical flexibility to optimize performance. The findings suggest that future iterations of the system could benefit from more comprehensive modeling and advanced control techniques to further enhance resource efficiency and operational reliability.

# Conclusions and Recommendations

The FalconSAT-9 attitude control system successfully achieved the mission objectives for both Control Mode 0 and Control Mode 1. The detumbling process stabilized the spacecraft within the required timeframe, leveraging nonlinear control techniques and deadband optimization to prevent limit cycles. The linear quadratic regulator demonstrated exceptional performance in maintaining precise orientation and handling external disturbances. These results validate the feasibility of the proposed control strategies.

For future work, it is recommended to refine the system model to include higher-fidelity representations of actuator dynamics and environmental disturbances. Additional optimization of the control law parameters could further enhance system efficiency and performance. Expanding the simulation to include multi-axis coupling effects may also yield valuable insights into potential improvements. These enhancements will ensure continued success in future spacecraft missions requiring advanced attitude control.

# Appendices

### Appendix A

Include supplementary materials, calculations, and MATLAB Simulink block diagrams here.

### Appendix B

### Appendix C

### Appendix D